



الزمن 3 ساعات

الامتحان مكون من (5) أسئلة في صفحة واحدة و المطلوب الإجابة عن كل الأسئلة

Marks

[1](a) Test the series: (i)  $\sum_{n=1}^{\infty} \frac{n^2}{3^n + 2}$       (ii)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4 + 3n}$       (6)

(b) Determine the interval of convergence of the series  $\sum_{n=0}^{\infty} \frac{(x-2)^n}{2^n}$       (6)

(c) If  $f(x, y, z) = x^2 + 3yz + 2z^2$ . Show that  $xf_x + yf_y + zf_z = 2f$       (6)

[2](a) If  $\bar{U} = 3i - 2j + 2k$  and  $\bar{V} = i + 4j + 2k$ . Find  $\bar{U} + \bar{V}$ ,  $\bar{U} \cdot \bar{V}$ ,  $\bar{U} \times \bar{V}$       (6)

(b) If  $\bar{U} = (y \sin x)i + (z \cos y)j + (xyz)k$ . Find  $\nabla \cdot \bar{U}$  and  $\nabla \times \bar{U}$ .      (6)

(c) Find the curvature of the curve  $x = t^3 + 2t - 1$ ,  $y = t^2 + \ln t$  at  $t = 1$       (6)

[3](a) Find the curvature and the circle of curvature of the curve  $y = x^4 + 2x - 1$  at the point (1, 2).      (8)

(b) Find the maximum and minimum values of the function:

$f(x, y) = x^2 - xy^2 + y$       (6)

(c) Find the extrema of  $f(x, y, z) = x^2 + y^2 + z^2$  subject to  $x + 2y + 3z = 28$       (6)

[4](a) Find the envelope of the curves  $x \sin a + y \cos a = 1$ ,  $a$  is parameter.      (6)

(b) Solve the equation  $(x + \sin y) dy + (y + \cos x) dx = 0$       (6)

(c) Find the solution of the equation  $(D^3 + D)y = \sin x$       (8)

[5] Solve the following differential equations:      (24)

(a)  $y' + \frac{4}{x}y = \frac{1}{x}$       (b)  $y'' + 9y = x^4 + e^{4x}$

(c)  $y'' - 6y' + 9y = 3 + e^{3x}$       (d)  $(D^2 + 1)y = \tan x$